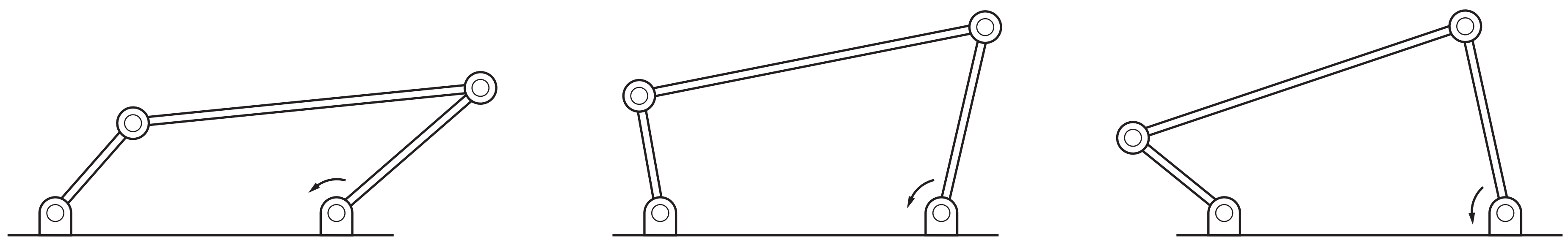
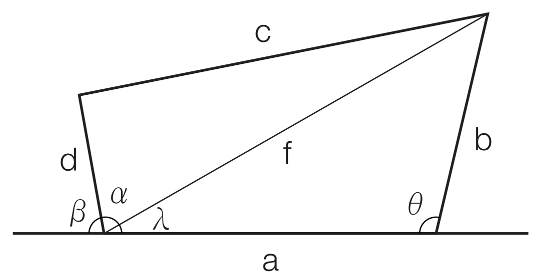
Four Bar Linkage Math

objective: Generate an expression that determines the rotation amount of the left four bar link knowing the rotation amount of the right link, the length of all the links, and the distance between the bottom pivots of the left and right links.





Known:

a is the distance between the bottom pivots of the left and right links.
b, c, and d are the lengths of the three links.
θ is the rotation angle of link b relative to ground

$\beta = 180 - (\alpha + \lambda)$
$\beta = 180 - \alpha - \lambda$

Law of cosines:

$f^2 = a^2 + b^2 - 2ab\cos\theta$
$b^2 = a^2 + f^2 - 2af\cos\lambda$
$c^2 = d^2 + f^2 - 2df\cos\alpha$

Substitute for f to find λ and α in terms of what is known (a, b, c, d, θ):

$$b^2 = a^2 + a^2 + b^2 - 2ab\cos\theta - 2a\sqrt{a^2 + b^2 - 2ab\cos\theta}\;\cos\lambda \longrightarrow \lambda = \cos^{-1}\left(\frac{-2a^2 + 2ab\cos\theta}{-2a\sqrt{a^2 + b^2 - 2ab\cos\theta}}\right)$$

$$c^2 = d^2 + a^2 + b^2 - 2ab\cos\theta - 2d\sqrt{a^2 + b^2 - 2ab\cos\theta}\;\cos\alpha \longrightarrow \alpha = \cos^{-1}\left(\frac{c^2 - d^2 - a^2 - b^2 + 2ab\cos\theta}{-2d\sqrt{a^2 + b^2 - 2ab\cos\theta}}\right)$$

Substitute for λ and α in terms of what is known (a, b, c, d, θ):

$$\beta = 180 - \cos^{-1}\left(\frac{c^2 - d^2 - a^2 - b^2 + 2ab\cos\theta}{-2d\sqrt{a^2 + b^2 - 2ab\cos\theta}}\right) - \cos^{-1}\left(\frac{-2a^2 + 2ab\cos\theta}{-2a\sqrt{a^2 + b^2 - 2ab\cos\theta}}\right)$$